15. $f(x, y, z) = xe^y + ye^z + ze^x$, $(0, 0, 0)$, $\mathbf{v} = \langle 5, 1, -2 \rangle$

$\nabla f \cdot u_v$

$= \langle e^y + \cancel{z}e^x, x\cancel{e^y} + e^z, \cancel{ye^z} + e^x \rangle \cdot \langle \frac{5}{\sqrt{30}}, \frac{1}{\sqrt{30}}, \frac{-2}{\sqrt{30}} \rangle \Big|_{(0,0,0)} =$

$= \frac{5}{\sqrt{30}} + \frac{1}{\sqrt{30}} - \frac{2}{\sqrt{30}} = \frac{4}{\sqrt{30}}$

31. The temperature T in a metal ball is inversely proportional to the distance from the center of the ball, which we take to be the origin. The temperature at the point (1, 2, 2) is 120°.

(a) Find the rate of change of T at (1, 2, 2) in the direction toward the point (2, 1, 3).

(b) Show that at any point in the ball the direction of greatest increase in temperature is given by a vector that points toward the origin.

$y = \frac{k}{x}$

$T(x,y,z) = \frac{k}{\sqrt{x^2+y^2+z^2}}$, $k \to 360$

$T(x,y,z) = \frac{360}{\sqrt{x^2+y^2+z^2}}$

a) $\nabla T = \left\langle \frac{-360 \cdot x}{(x^2+y^2+z^2)^{3/2}}, \frac{-360 \cdot y}{(x^2+y^2+z^2)^{3/2}}, \frac{-360 \cdot z}{(x^2+y^2+z^2)^{3/2}} \right\rangle \Big|_{(1,2,2)}$

$= \left\langle -\frac{40}{3}, -\frac{80}{3}, -\frac{80}{3} \right\rangle$

$u = \left\langle \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right\rangle$

$\nabla T \cdot u = \frac{-40}{3\sqrt{3}} \cancel{+\frac{80}{3\sqrt{3}}} \cancel{-\frac{80}{3\sqrt{3}}} = -\frac{40}{3\sqrt{3}}$

b) greatest change $\to \nabla T$

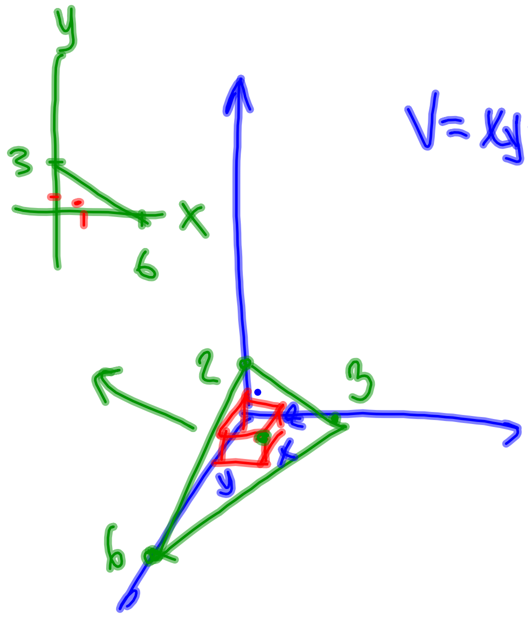
$\nabla T = \left\langle \frac{-360x}{(x^2+y^2+z^2)^{3/2}}, \frac{-360y}{(x^2+y^2+z^2)^{3/2}}, \frac{-360z}{(x^2+y^2+z^2)^{3/2}} \right\rangle$

$= \frac{360}{(x^2+y^2+z^2)^{3/2}} \langle -x, -y, -z \rangle$

$\max(\nabla f \cdot u_a) = \max(|\nabla f||u| \cdot \cos\theta)$

$= |\nabla f|$

47. Find the volume of the largest rectangular box in the first octant with three faces in the coordinate planes and one vertex in the plane $x + 2y + 3z = 6$.



$$V = xyz = xy\left(\frac{6-2y-x}{3}\right)$$

$$V_x = y\left(\frac{6-2y-x}{3}\right) + xy\left(-\frac{1}{3}\right) = 0$$

$$6y - 2y^2 - xy - xy = 0$$

$$6 - 2y = 2x$$

$$\boxed{3 - y = x}$$

$$V_y = x\left(\frac{6-2y-x}{3}\right) + \left(-\frac{2}{3}\right)xy = 0$$

$$6 - 2y - x - 2y = 0$$

$$\boxed{6 - 4y = x}$$

$$3 - y = 6 - 4y$$

$$y = 1$$

$$x = 2$$

$$V_{xx}V_{yy} - (V_{xy})^2 > 0$$